Homework #3
EE 232 – Lightwave devices (Spring 2019)

DUE: 3/07/2019 (Please hand-in homework prior to the beginning of lecture)

(1) In this problem, you will investigate bulk absorption and gain in indium phosphide (InP) at T=4K and T=300K. You can neglect the temperature dependence on the bandgap and assume E_g = 1.344eV for both temperatures. Further, you may assume $m_e = 0.077m_0$, $m_h = 0.6m_0$, $n_r = 3.5$ and E_p = 20.7 eV.

  a. Estimate the transparency carrier density (n_{tr}) for bulk InP at T=300K. Assume only the heavy-hole band is populated in the valence band.
  b. Plot the gain spectrum (gain as a function of photon energy) for injected carrier density of $n = 0.2n_{tr}$, $n = n_{tr}$, $n = 2n_{tr}$, and $n = 4n_{tr}$ at T=4K and T=300K. Assume only transitions between the conduction band and heavy-hole band. You will have to calculate the quasi-Fermi levels using the Fermi-Dirac integral. Refer to equation (38) in the paper by Blakemore for an approximation to the Fermi-Dirac integral.

(2) In this problem, you will investigate bulk absorption and gain in an AlGaAs/GaAs quantum well at T=0K. You can assume the GaAs bandgap to be E_g = 1.519 eV (at T=0K) and the quantum well thickness to be L_z = 8 nm. Further, the material parameters for GaAs are $m_e = 0.067m_0$, $m_{hh} = 0.5m_0$, $m_{lh} = 0.087m_0$, $n_r = 3.5$, and E_p = 25.7 eV. Assume the light traveling in the quantum well is transverse electric (TE).

  a. Calculate the energy at the bottom of the first conduction subband (E_{e1}), and top of the light-hole (E_{lh1}) and heavy-hole subbands (E_{hh1}). As a simplification, you may use the infinite barrier model.
  b. Write a general expression Fermi level splitting (F_c-F_v) as a function of carrier density for a quantum well at T=0K. Assume only the heavy-hole band is populated.
  c. Plot the gain spectrum of the quantum well (gain as a function of photon energy) for Fermi level splitting of $F_c-F_v=0$, and $F_c-F_v=1.02(E_g+E_{e1}+|E_{hh1}|)$. Plot the spectra such that the transitions between each band can be observed on the chosen x-axis scale. Assume only transitions between the first quantized conduction, heavy-hole, and light-hole subbands (i.e. n=m=1) and assume the overlap integral is equal to unity.

(3) Show that, for a cube-like quantum dot with infinite barriers, the momentum matrix element is given by

$$\hat{e}\cdot\mathbf{p}_{cv} = \langle \psi_c | \hat{e}\cdot\mathbf{p} | \psi_v \rangle = \delta_{m,m'}\delta_{n,n'}\delta_{l,l'} \langle u_c | \hat{e}\cdot\mathbf{p} | u_v \rangle$$

where ψ_c and ψ_v are the Bloch states in the conduction and valence band, u_c and u_v are the Bloch functions, $\mathbf{p}$ is the momentum operator, δ is the Kronecker delta function where the indices correspond to the quantum number of the confined state in the x,y, and z directions.